Quiz 3, MATH 240, Fall 2023

Write your name clearly.

Name:

Section Number: UID:

(1) Let A be a matrix that has the RREF

$$\begin{pmatrix} 1 & 2 & 0 & -1 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(a) [8] Write the parametric vector form of the solution set to $A\mathbf{x} = \mathbf{0}$.

(b) [8] If we know $A\begin{pmatrix} 2 \\ -1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$, write the parametric vector form of the solution set to $A\mathbf{x} = \begin{pmatrix} 2 \\ 2 \\ 2 \\ 2 \end{pmatrix}$.

(c) [4] Are the columns of A linearly independent or linearly dependent? Explain.

(a) The solution is

$x_1 + 2x_2 - x_4 = 0$

$x_3 - 4x_4 = 0$

x_2, x_4 free,

So the parametric vector form of the solution set is

$$\vec{x} = x_2\begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_4\begin{pmatrix} 1 \\ 0 \\ 4 \\ 1 \end{pmatrix}.$$

(b) Note that $A(4,-2,0,2) = (2,2,2,2)$. So the solutions to $A\vec{x} = (2,2,2,2)$ are given by

$$\vec{x} = (4,-2,0,2) + x_2(-2,1,0,0) + x_4(1,0,4,1).$$

(c) The columns of A are linearly dependent because the RREF of A does not have a pivot in every column.